CLASSICAL APPROACH TO BRITTLE FRACTURE GROWTH

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ABSTRACT

In this current scenario the usage of composite material is under rapid development in the field of industrial, Aerospace, Marine industries. We meet with difficulty of using the composite due to the formation of cracks. Cracks in composites usually appear during the stage of manufacture or in very earlier stage of use. In this paper, Fracture Mechanics has evolved as a result of attempts, to understand and prevent such failures. Here a mathematical model for brittle fracture in a fibre composite along transverse crack growth is given. The probability of spontaneous growth of an initial crack and subcritical crack growth is the solution of a functional equation satisfied by probability of events based on crack growth.

Key words: Fibre composite, brittle fracture, transverse crack growth, functional equation, orthotropic body, Markov chain.

1. INTRODUCTION

Brittle fracture is fracture that involves little or no plastic deformation. It is usually associated with flaws or defects in the material where bulk stresses concentrate. A stress intensity is associated with flaws and a stress concentration factor can be assigned to the flaw or notch based on its geometry, location and orientation. Brittle fracture is one of the major concerns in Industrial Damage Mechanism and characterized by a sudden fracture in the material under stress. Normally all materials of construction are susceptible to damage and efforts have been made to prevent failures in equipment caused by fracture mechanism. The reason for brittle fracture is the inherent variability in size, shape, orientation and nature of molecular, microscopic and macroscopic defects. Brittle fracture may occur under various types of loading either in short term loading or long term loading. The brittle fibers used in composites have a high coefficient of variation of strength typically 5-30% and a much higher coefficient of variation of lifetime under sustained loading. Size effects may be defined as longitudinal or transversal effects if the roles of length and width are dealt with. It has the possibility of producing very high strength polymers in fibrous or whiskers form.

2. BASIC DEFINITIONS

Markov chain: The Stochastic process $\{X_n : n = 0,1,2,\ldots\}$ is called a Markov chain, if, for j, k, $j_1, \ldots, j_{n-1}$ in N (for any subset of I), $P_r\{X_n = k / X_{n-1} = j, X_{n-2} = j_1, \ldots, X_0 = j_{n-1}\} = Pr\{X_n = k / X_{n-1} = j\} = p_{jk}$ say.

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The outcomes are called the states of the Markov chain; if X_n has the outcome j (i.e., X_n=j), the process is said to be at state j at n_{th} trial. To a pair of states (j,k) at the two successive trials (say, n_{th} and $n+1_{th}$ trials), there is an associated conditional probability p_{ik}. It is the probability of transition from the state j at nth trial to the state k at $n+1_{th}$ trial. The transition probability p_{jk} are basic to the study of the structure of the Markov chain.

Fibre composites: Fibre composites involve two components, namely filament and a bonding agent which ties fibre filaments.

Orthotropic body: Orthotropic materials have properties that are different in three mutually perpendicular directions. They have three mutually perpendicular axes of symmetry, and load applied parallel to these axes produces only normal strains. However, loads that are not applied parallel to these axes produce both normal and shear strains. Therefore, orthotropic mechanical properties are a function of orientation.

Assumption: Consider a strip model of a composite when all of the fibre cylinders of radius r_0 are placed with parallel axes in one plane in a strip of binder of width 'h' at identical distance 'd' from each other. The composite is uniformly stretched in the direction of fibres by stress σ = $E_1 \in_\infty$. The placement of the fibres are located in the stress concentration zone near the tip of a crack, the size of this zone along the axes of the fibres is equal to the ineffective length L.

3. MAIN RESULTS

Proposition 1

To find the stresses in whole fibre located near the lost broken fibre.

Proof

Let us suppose a composite has an initial crack of length $2l_0$. We consider that the plate can be replaced by a homogeneous orthotropic plate of equivalent elastic reaction (with thickness h) with axis of elastic symmetry along and across the fibres. The stress on the continuation of the transverse through crack in such a plate is where k_1 is the stress intensity, x is the distance from the tip of the crack along the continuation. Using the conditions of equilibrium

$$\sigma_z \pi r_0^2 = h \int_{(k-1)d}^{kd} \frac{k_1}{\sqrt{2\pi x}} dx$$

$$= \frac{k_1 h}{\sqrt{2\pi}} \int_{(k-1)d}^{kd} \frac{dx}{x}$$

$$\boldsymbol{\sigma}_z = \frac{k_1 h}{\pi\sqrt{\pi r_0^4}} \sqrt{2d}(\sqrt{k} - \sqrt{k-1})$$

K_d is the distance to the centre of the fibre in question from the tip of the crack. In particular in most highly stressed, nearest fibre(i.e., when k=1) we have

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$$\boldsymbol{\sigma}_z = \frac{k_1 h}{\pi\sqrt{\pi r_0^4}}\sqrt{2d}$$

The growth of a crack occurs by successive breaking of the most highly stressed fibres near the crack tip, while other possible breaks within the depth of the body will be ignored. In the case of isolated crack we have $k_1 = \sigma\sqrt{\pi l}$ where 2l is the length of the crack. In other cases k_1 is determined by the methods of the theory of elasticity for an equivalent orthotropic body.

Proposition 2

To find the probability of spontaneous growth of an initial crack under constant external load.

Proof:

Let A_n be the event length of crack equal to $2(l_0+n_d)$.

$$\Pr\{A_n / A_{n+1}\} = \int_0^{\sigma_n} p(\sigma)d\sigma$$

$$= 1- \exp\left(-\alpha L\sigma_n^{\beta}\right)$$

Where in the case of an isolated crack

$$\sigma_n = \frac{\sigma_h\sqrt{2d}}{\pi r_0^2}\sqrt{l_0 + n_d}$$

In the case of other geometric configurations of the body and crack, the stress configuration σ_n is found by

$$\frac{k_1 h\sqrt{2d}}{\pi\sqrt{\pi r_0^4}}$$

Proposition 3

The probability of spontaneous growth of an initial crack is $p(\sigma, l_0) =$

$$\prod_1^{\infty} \boldsymbol{p}\ \boldsymbol{P}(\boldsymbol{A}_{\boldsymbol{n}+1} / \boldsymbol{A}_{\boldsymbol{n}})$$

Proof

Let us consider a certain event impossible if its probability is less than P_c. The value of P_c should be assigned as a function of the purpose and importance of structure. From this assuming $p(\sigma, l_0)P_c$, we can find safe region on the plane of parameters σ and l_0.

In particular, if all fibres have identical strength σ_f

That is $p(\sigma) = \delta(\sigma - \sigma_f)$, the region is bounded by the following formula

$k_1 < \frac{\pi r_0^2 \sigma_f \sqrt{\pi}}{h\sqrt{2d}}$ for an isolated crack $k1 = \sigma\sqrt{\pi l}$.

$$\text{Therefore } \sigma < \frac{\pi r_0^2 \sigma_f}{h\sqrt{2dl_0}}$$

The probability of spontaneous fracture of a defect – free strip composite under constant load is $P(\sigma) = p_0 = \prod_1^{\infty} p_n$, l_0=0,

$$\text{Where } p_0 = \int_0^{\sigma} p(\sigma)d\sigma = 1 - \exp\left(-L\alpha\sigma^{\beta}\right)$$

Proposition 4

Consider the subcritical crack growth. To study the discrete random process of formation and growth of an edge transverse linear crack in a strip of width N_d, where N is the number of fibres in the strip). The external load σ increases always by the quantity Δ.

Proof

The tip of a crack moves along a straight line perpendicular to the boundaries of the strip to the right under the influence of random breakage of fibre occurring at loads 1Δ, 2Δ,3Δ,…,kΔ.

The tip of the crack may be at points with coordinates 1d,2d,3d,…,Nd. As the load increases from kΔ to (k+1) Δ, the tip of the crack with probability α_{nks} moves to the right from position nd. By distance sd to position (n+s)d, when nd is the coordinate of the tip of the crack under load kd, and s is the number of fibres broken as the load is increased by Δ. The crack length remains equal to nd under load (k+1) Δ with probability.

$$\beta_{nk} = 1 - \sum_{s=1}^{n-x} \alpha_{nks}$$

Proposition 5

Let us assume that fibres fracture only at the front of the crack, internal fractures will be ignored. P_{nk} = Probability that under load kΔ the length of crack is equal to nd.

Proof

The value of the stress intensity factor k_1 for the edge crack of length nd is an orthotropic strip width Nd is approximately equal to intensity factor k_1 for a periodic system of cracks of length 2nd along the x –axis with period 2[nd] (with the same tensile stress at infinites). This equation is fulfilled more exactly the greater the ratio of young's modulus along the fibres to young's modulus across the fibres. From this using the formula

$$k_1 = \sigma\sqrt{2b\tan\left(\frac{\pi c}{2b}\right)} + \frac{p}{\sqrt{b\sin\left(\frac{\pi c}{b}\right)}}$$

Where l = nd and σ = (k+1)Δ

According to formula

$$\sigma_z = \frac{k_1 h}{\pi\sqrt{\pi r_0^4}}\sqrt{2d}$$

We can calculate the stress in the most highly stressed fibre near the tip of the crack

$$\sigma_z = \gamma_{nk} = \frac{2\Delta nd}{\pi\sqrt{\pi r_0^2}}\,(k+1)\sqrt{N\tan\left(\frac{\pi n}{2N}\right)}$$

Proposition 6

Now, utilizing the weibull distribution and formula for conditional distribution. We can easily find the transitions α_{nks}.

Proof

The analytic solution of this equation yielding expressions of p_{nk} for all values of n and k, is too cumbersome and inconvenient for us. It should be noted that the functional equation assuming the interval breakage of fibres in the process of growth is so complex and unmanageable that it becomes quite useless. With a sufficiently small load step Δ, we can approach continuous loading σ(t) arbitrarily closely. We construct the limiting load σ_b as a functional critical crack length l_c, as well as the probability density $p_0(\sigma_b)$ of random quantity σ_b. After this end desired limiting load is found as the mathematical expectation of random quantity σ_b.

$E(\sigma_b) = \sum p_0(\sigma_b)$.

The problem is identical to one arising in the theory of non-homogeneous Markov chains. Suppose a certain particle moves along a straight line to the right the influence of random impulses occurring at moments in time 1,2,3,…,k,… The particle may be located at points with coordinates 0,1,2,3,…,N. Each impulse instantaneously displaces the particle by a distance to the right with assigned probability α_{nks} with probability β_{nk}, the particle remains at its previous position. At the initial movement k=0, the particle was located at point n=0 with p_{00}. The problem is to find p_{nk}. The process end as soon as the particle reaches point n=N.

Solution is given as follows: The tip of the crack may arrive at point nd under load kΔ only by the following (n+1) mutually exclusive means (0) under load (k-1) Δ remained at nd as the load was increased (1) under load (k-1) Δ it was located at point (n-r)d where r=1,2,3,…,n.
Adding the probabilities of the corresponding means we arrive at the following functional equation for the process $P_{nk} = p_{n,k-1}\alpha_{n,k-1} + p_{n-1,k-1}\alpha_{n-1,k-1,1} + \ldots + p_{n-r,k-1}\alpha_{n-r,k-r,r} + \ldots + p_{0,k-1}\alpha_{0,k-n,n}$. The problem is to solve the functional equation in the region n>0, k>0 with the following boundary conditions

$P_{0,0} = P_{1,0} = \ldots = P_{N,0} = 0$, $P_{0.k} = 1- e^{(\alpha;lk\Delta\beta)}$, k= 1,2, …, k.

Using the Weibull distribution

$$\int_0^{\sigma_f} p(\sigma)d\sigma = 1 - \exp(-\alpha l\sigma_f\ \beta)$$

The probability of breakage of an edge fibre under load Δ. The quantity $P_{0,k}$ represent the probability that the fibre adjacent to its will not break under load σ=kΔ. In order to determine the stress in this fibre, we have made the assumption that the entire increase in load due to the failure of the edge fibre is received by neighbouring fibre alone. This assumption is beyond doubt in the case when young's modulus of the fibres is much greater than of the matrix. When l>2d, in order to calculate the concentration of stresses in the most highly stressed fibre at the tip of the crack, we can use the method of the effective orthotropic body and formula for σ_z.

The theory we have presented is easy to extend to the case when a composite strip of finite width has an initial edge crack of length $l_0 = n_0 d$. In this case the solution of the same functional equation can be easily found in the region $n>n_0$, K>0 under the following conditions:

$P_{nk} = 0$ for $p_{n_{0,0}} = 1$, $p_{n_{0,1}} = \exp\left(-\alpha L\gamma_{n_0}^{\beta}, 0\right)$, $p_{n_{0,2}} = \exp\left(-\alpha L\gamma_{n_0}^{\beta}, 1\right)$,.,.,

$p_{n_{0,k}} = \exp\left(-\alpha L\gamma_{n_0}^{\beta}, k-1\right)$

Where $\gamma_{n,k}$ is given by the formula

$$\gamma_{n,k} = \frac{2\Delta hd}{\pi\sqrt{\pi r_0^2}}\sqrt{N\,tan\left(\frac{\pi x}{2N}\right)}$$

The effect of the derived crack growth is formally identical to subcritical growth of through cracks in plates of an elastic plastic material with an increase in load, however as we can see it has quite different physical nature. In particular with repeated loading to the previous level of load, the formation of new cracks (and the growth of old cracks) will not occur in this case. The theory can be with almost no changes, transferred to arbitrary structure's of unidirectional fibre composite with sufficiently dense placement of fibres.

CONCLUSION

Fatigue crack growth and brittle fracture play an important role in design and fabrication of composite materials used in various engineering applications. Methods based on differential equations and material mechanics cannot give the exact reasons for crack in composite materials. This is because of uncertainty prevalent in structure of composite materials. So, probabilistic approach is needed to study such problems, we have given Stochastic Markov chain discussion leading to a functional equation to be solved by Stochastic functional equation.

REFERENCES

1. Bharucha – Reid, A.T., 'Elements of Markov processes and their applications', McGraw Hill, New York, 1960.
2. Cherepancy,G.P.,, 'Mechanics of Brittle Fracture, McGraw Hill Company, U.K. London, 1979.
3. Doob, J.L., Stochastic Processes', John Wiley, New York, 1953.
4. Zygmund, A, 'Trigonometric Series', Cambridge University Press, London, 9160.
5. Scop, P.M., and Argon, A.S., 'Statistical theory of strength of laminated composites', Int.J. Composite Materials, Vol. 1, 3, 1969, p.30.
6. Broutman, L.J. and Krock, R.H. (Eds), 'Composite Materials' in 8 vols(Especially Vol 5 'Fracture and Fatigue'). Academic press, New York London, 1974.
7. Thiyagarajan, M., 'Stochastic analysis of some problems in social and biological sciences' , A Ph.D. thesis submitted to University of Madras, March 1984.

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Born in 1940 and brought up in the southern most state of Tamil nadu, India. Dr. M. Thiyagarajan has almost spent his life as a researcher and teacher on account of his passion and affinity towards applied mathematics Dr. M. Thiyagarajan initiated his educational career on Computer Science in both under graduate and post graduate programmes. He has successfully fulfilled his ambition and desire by receiving his doctoral degree in Applied Mathematics from the time tested and age old University of Madras, Chennai in 1985.

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