MATHEMATICAL MODEL FOR THE TRANSMISSION OF AVIAN INFLUENZA BY AGE GROUP OF PATIENTS IN THAILAND

Puntani Pongsumpun (Corresponding author) & **Jiraporn Lamwong**
Department of Mathematics, Faculty of Science, King Mongkut's
Institute of Technology Ladkrabang, Chalongkrung road, Ladkrabang, Bangkok 10520
THAILAND

ABSTRACT

Avian influenza is caused by influenza virus type A, which is called H5N1. In 2004, an epidemic was recognized as the first time in Thailand. After that, there were the reports of the sporadic outbreaks in all regions. This disease can be transmitted to human by birds. Human can be infected by direct contact from infectious animals by touching the phlegm or biological fluid contact with the feces of infectious animals. In this study, we take into account the age structure of avian influenza patients. We separated the population into two groups such as human and birds. Age structure of human population is separated into two classes; juvenile and adult human. The equations are constructed for each class. Standard dynamical modeling method is used for analyzing the behaviors of solutions. The stability conditions for the disease free equilibrium state and disease endemic equilibrium states are determined. The basic reproductive number is found. The numerical solutions are shown for supporting the theoretical results and we analyze method for controlling the transmission of avian influenza. The results of this study suggest the way for reducing the outbreak of this disease.

Keywords: Basis Reproductive Numbers, Disease Free Steady State, Endemic Steady State, Stability.

INTRODUCTION

Avian influenza is caused by influenza virus type A, which is called H5N1. Avian influenza is an infectious disease of birds (especially water fowl such as ducks and birds), virus can spread to domestic poultry and cause large-scale outbreaks of serious diseases. Some of these H5N1 viruses have also been reported to cross the species barrier and cause disease or subclinical infections in humans and other mammals. Viruses are separated into 2 groups based on their abilities to cause disease in poultry: high pathogenicity or low pathogenicity. Highly pathogenic viruses result in high death rates (up to 100% mortality within 48 hours) for some poultry species. Low pathogencity viruses also cause outbreaks in poultry but they are not generally associated with severe disease (World Health Organization, 2011). First infected humans were reported in 1997 during a poultry outbreak in Hong Kong SAR (World Health Organization, 2012), China. Since its widespread re-emergence in 2003 and 2004, this avian virus has spread from Asia to Europe and Africa. It has become entrenched in poultry in some countries, resulting in millions of poultry infections, several hundred human cases, and many human die from this disease. Outbreaks in poultry have seriously impacted livelihoods, the economy and international trade in affected countries. From the above mentioned, we need to find the way for reducing the outbreak of Avian Influenza. The data of patients collected from Ministry of public health, Thailand indicated that there are the different transmission rates between juvenile and adult.

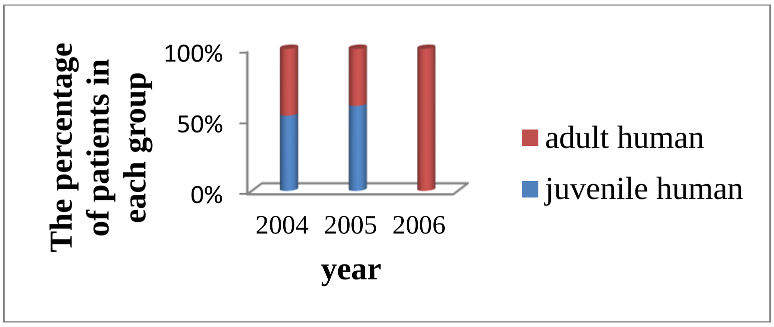


Fig1. The data of Thai influenza cases of Thailand. There is no data in year 2007

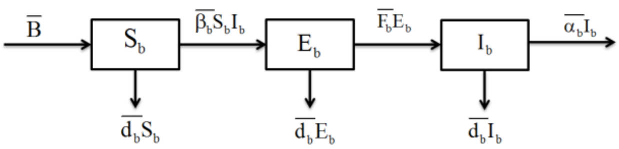
In 2008, Mohamed and Abdesslam (Mohamed, D., & Abdesslam, B., 2008) studied the dynamics of human who be infected by avian influenza, they presented a mathematical model and show the stability analysis and simulations with the different parameters. In 201, chong Tchuenche and Smith (Nyuk Sian Chong, Jean Michel Tchuenche, & Robert J. Smith, 2013) studied the half-saturated incidence rate $\frac{\beta SI}{H+I}$. The parameter $\beta > 0$ is the transmission rate and H is the half-saturation constant, i.e., the density of infected individuals in the population that yields 50 % possibility of contracting avian influenza. In this paper, we studied the transmission of Avian influenza virus by formulating the mathematical model of avian influenza for bird and human populations. The two steady states are obtained, conditions for stabilities of disease free and endemic steady states were investigated and showed in the form of basis reproductive numbers. The mathematical solutions are shown to support the theoretical solutions.

FORMULATION OF THE MODEL

In this study, we consider the transmission of avian influenza. For bird, we separate into three types: susceptible, exposed and infected groups. For human, we divide into 8 groups; susceptible, exposed, infected, recovered juvenile humans, susceptible, exposed, infected and recovered adult humans.

The transmission diagrams of bird and human populations are shown in fig.2.

B population



Human population

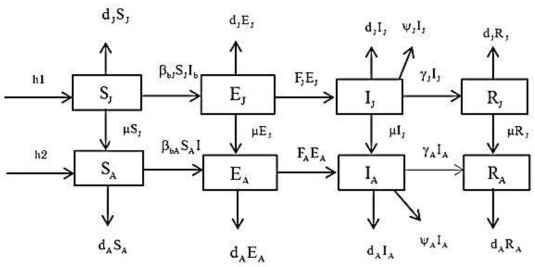


Fig.2 Diagram of our model

We define the variables and parameters in the model as follows:

$S_b(t)$ is the number of susceptible birds at time t,
$E_b(t)$ is the number of exposed birds at time t,
$I_b(t)$ is the number of infected birds at time t,
$S_J(t)$ is the number of susceptible juvenile human at time t,
$E_J(t)$ is the number of exposed juvenile human at time t,
$I_J(t)$ is the number of infected juvenile human at time t,
$R_J(t)$ is the number of recovered juvenile human at time t,
$S_A(t)$ is the number of susceptible adult human at time t,
$E_A(t)$ is the number of exposed adult human at time t,
$I_A(t)$ is the number of infected adult human at time t,
$R_A(t)$ is the number of recovered adult human at time t.

The dynamical equations are described as follows:

Bird population

$$\frac{dS_b}{dt} = \bar{B} - \bar{\beta}_b S_b I_b - \bar{d}_b S_b \qquad (1.1)$$

$$\frac{dE_b}{dt} = \bar{\beta}_b S_b I_b - (\bar{F}_b + \bar{d}_b) E_b \qquad (1.2)$$

$$\frac{dI_b}{dt} = \bar{F}_b E_b - (\bar{\alpha}_b + \bar{d}_b) I_b \qquad (1.3)$$

Human population

$$\frac{dS_J}{dt} = h1 - \beta_{bJ} S_J I_b - (\mu + d_J) S_J \qquad (1.4)$$

$$\frac{dE_J}{dt} = \beta_{bJ} S_J I_b - (F_J + \mu + d_J) E_J \qquad (1.5)$$

$$\frac{dI_J}{dt} = F_J E_J - (\mu + \gamma_J + \psi_J + d_J) I_J \qquad (1.6)$$

$$\frac{dR_J}{dt} = \gamma_J I_J - (\mu + d_J) R_J \qquad (1.7)$$

$$\frac{dS_A}{dt} = h2 - \beta_{bA} S_A I_b + (\mu - d_A) S_A \qquad (1.8)$$

$$\frac{dE_A}{dt} = \beta_{bA} S_A I_b + \mu E_J - (F_A + d_A) E_A \qquad (1.9)$$

$$\frac{dI_A}{dt} = \mu I_J + F_A E_A - (\gamma_A + \psi_A + d_A) I_A \qquad (1.10)$$

$$\frac{dR_A}{dt} = \mu R_J + \gamma_A I_A - d_A R_A \qquad (1.11)$$

where the parameters are defined in table1.

Table 1. The definitions of parameters for our model

Symbol	Description
$\bar{B}$	bird inflow
$\bar{d}_b$	Natural death rate of birds
$\bar{\alpha}_b$	death rate due to avian strain in birds
$\bar{\beta}_b$	rate at which susceptible bird change to be exposed bird
$\bar{F}_b$	incubation rate of avian influenza in birds
h1	juvenile human recruitment rate
β_{bJ}	transmission rate of avian influenza from birds to juvenile human population

F_J	incubation rate of avian influenza in juvenile human population
γ_J	recovery rate of juvenile human
ψ_J	death rate due to avian strain in juvenile human
d_J	Natural death rate of juvenile human
μ	rate at which juvenile change to be adult human population
h2	adult human recruitment rate
β_{bA}	transmission rate of avian influenza from birds to adult human population
F_A	incubation rate of avian influenza in adult human population
γ_A	recovery rate of adult human
ψ_A	death rate due to avian strain in adult human
d_A	Natural death rate of adult human

ANALYSIS OF THE MATHEMATICAL MODEL
Equilibrium Points

Setting (1.1)-(1.11) to zero, then the equilibrium points are given by:

i) The disease free state: $E_1 = (\frac{\bar{B}}{\bar{d}_b}, 0, 0, \frac{h1}{d_J+\mu}, 0, 0, 0, \frac{d_J h2 + (h1+h2)\mu}{d_A(d_J+\mu)}, 0, 0, 0)$

ii) The endemic disease state: $E_2 = (S_b^*, E_b^*, I_b^*, S_J^*, E_J^*, I_J^*, R_J^*, S_A^*, E_A^*, I_A^*, R_A^*)$

$$S_b^* = \frac{\bar{B}}{\bar{\beta}_b I_b^* + \bar{d}_b}$$

$$E_b^* = \frac{\bar{B}\bar{\beta}_b I_b^*}{(\bar{F}_b + \bar{d}_b)(\bar{\beta}_b I_b^* + \bar{d}_b)}$$

$$I_b^* = \frac{\bar{B}\bar{F}_b}{(\bar{d}_b + \bar{F}_b)(\bar{d}_b + \bar{\alpha}_b)} - \frac{\bar{d}_b}{\bar{\beta}_b}$$

$$S_J^* = \frac{h1}{\beta_{bJ} I_b^* + (\mu + d_J)}$$

$$E_J^* = \frac{h1\beta_{bJ} I_b^*}{(F_J + \mu + d_J)(\beta_{bJ} I_b^* + \mu + d_J)}$$

$$I_J^* = \frac{h1\beta_{bJ} F_J I_b^*}{(\mu + \gamma_J + \psi_J + d_J)(F_J + \mu + d_J)(\beta_{bJ} I_b^* + \mu + d_J)}$$

$$R_J^* = \frac{h1\beta_{bJ} F_J \gamma_J I_b^*}{(\mu + d_J)(\mu + \gamma_J + \psi_J + d_J)(F_J + \mu + d_J)(\beta_{bJ} I_b^* + \mu + d_J)}$$

$$S_A^* = \frac{h1\mu + h2(\beta_{bJ} I_b^* + \mu + d_J)}{(\beta_{bA} I_b^* + d_A)(\beta_{bJ} I_b^* + \mu + d_J)}$$

$$E_A^* = \frac{I_b^*\left(\frac{h1\beta_{bJ}\mu}{d_J + F_J + \mu} + \frac{\beta_{bA}(h1\mu + h2(d_J + \beta_{bJ} I_b^* + \mu))}{d_A + \beta_{bA} I_b^*}\right)}{(d_A + F_A)(d_J + \beta_{bJ} I_b^* + \mu)}$$

$$I_A^* = \frac{\dfrac{F_A\left(\dfrac{h1\beta_{bJ}\mu I_b^*}{F_J + \mu + d_J} + \dfrac{\beta_{bA} I_b^*(h1\mu + h2(\beta_{bJ} I_b^* + \mu + d_J))}{\beta_{bA} I_b^* + d_A}\right)}{F_A + d_A} + \dfrac{h1\beta_{bJ} F_J \mu I_b^*}{(\mu + \gamma_J + \psi_J + d_J)(F_J + \mu + d_J)}}{(\gamma_A + \psi_A + d_A)(\beta_{bJ} I_b^* + \mu + d_J)}$$

$$R_A^* = \frac{\mu}{d_A}\left(\frac{h1\mu + h2(\beta_{bJ} I_b^* + \mu + d_J)}{(\beta_{bA} I_b^* + d_A)(\beta_{bJ} I_b^* + \mu + d_J)}\right) + \frac{\gamma_A}{d_A}\left(\frac{\dfrac{F_A\left(\dfrac{h1\beta_{bJ}\mu I_b^*}{F_J + \mu + d_J} + \dfrac{\beta_{bA} I_b^*(h1\mu + h2(\beta_{bJ} I_b^* + \mu + d_J))}{\beta_{bA} I_b^* + d_A}\right)}{F_A + d_A} + \dfrac{h1\beta_{bJ} F_J \mu I_b^*}{(\mu + \gamma_J + \psi_J + d_J)(F_J + \mu + d_J)}}{(\gamma_A + \psi_A + d_A)(\beta_{bJ} I_b^* + \mu + d_J)}\right)$$

Local Stability

The local stability of equilibrium point is determined by the sign of eigenvalues for each equilibrium state. If signs of the real parts all equilibrium points are negative, then that equilibrium point will be locally asymptotically stable.

- Let $E_1 = (S_b^*, E_b^*, I_b^*, S_J^*, E_J^*, I_J^*, R_J^*, S_A^*, E_A^*, I_A^*, R_A^*)$ .If $R_0 \le 1$ the disease free state E_1 is globally asymptotically stable in $\omega = \left\{ (S_b, E_b, I_b, S_J, E_J, I_J, R_J, S_A, E_A, I_A, R_A) \in \mathbb{R}_+^{11} : N_b \le \frac{\bar{B}}{\bar{d}_b}, N_J \le \frac{h1}{d_J}, N_A \le \frac{h2}{d_A} \right\}$.
- Let $E_2 = (S_b^*, E_b^*, I_b^*, S_J^*, E_J^*, I_J^*, R_J^*, S_A^*, E_A^*, I_A^*, R_A^*)$ .If $R_0 > 1$ the endemic diseases state E_2 is locally asymptotically stable in $\omega = \left\{ (S_b, E_b, I_b, S_J, E_J, I_J, R_J, S_A, E_A, I_A, R_A) \in \mathbb{R}_+^{11} : N_b \le \frac{\bar{B}}{\bar{d}_b}, N_J \le \frac{h1}{d_J}, N_A \le \frac{h2}{d_A} \right\}$.

i) Disease free state $E_1 = (\frac{\bar{B}}{\bar{d}_b}, 0, 0, \frac{h1}{d_J + \mu}, 0, 0, 0, \frac{d_J h2 + (h1 + h2)\mu}{d_A (d_J + \mu)}, 0, 0, 0)$,the characteristic equation is

$$\left| J_{E_1} - \lambda I_{11} \right| = 0$$

or

$$\begin{vmatrix}
-(\beta_{bJ} * I_b + d_J + \mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{bJ} * S_J \\
\beta_{bJ} * I_b & -(d_J + F_J + \mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{bJ} * S_J \\
0 & F_J & -(\gamma_J + \psi_J + d_J + \mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma_J & -(d_J + \mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu & 0 & 0 & 0 & -(\beta_{bA} * I_b + d_A) & 0 & 0 & 0 & 0 & 0 & -\beta_{bA} * S_A \\
0 & \mu & 0 & 0 & \beta_{bA} * I_b & -(d_A + F_A) & 0 & 0 & 0 & 0 & \beta_{bA} * S_A \\
0 & 0 & \mu & 0 & 0 & F_A & -(\gamma_A + d_A + \psi_J) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 & \gamma_A & -d_A & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\beta}_b * S_b - \bar{d}_b & 0 & -\bar{\beta}_b * S_b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{\beta}_b * S_b & -\bar{F}_b - \bar{d}_b & \bar{\beta}_b * S_b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{F}_b & -\bar{\alpha}_b - \bar{d}_b
\end{vmatrix} = 0$$

Then the characteristic polynomial of the above Jacobian matrix is

$$(-\lambda - d_A)^2(-\lambda - \mu - d_J)^2(-\lambda - d_A - F_A)(-\lambda - \mu - d_J - F_J)((-\lambda - \bar{d}_b - \bar{F}_b)(-\lambda - \bar{d}_b - \bar{\alpha}_b)(-\lambda - \bar{d}_b - \frac{\bar{B}\bar{\beta}_b}{\bar{d}_b}) - \bar{F}_b(-\bar{B}\bar{\beta}_b - \frac{\bar{B}\bar{\beta}_b \lambda}{\bar{d}_b}))(-\lambda - d_A - \gamma_A - \psi_A)(-\lambda - \mu - d_J - \gamma_J - \psi_A) = 0 \quad (3.1)$$

The eigenvalues are given by

$$\lambda_1 = -d_A, \lambda_2 = -\mu - d_J, \lambda_3 = -d_A - F_A, \lambda_4 = -\mu - d_J - F_J, \lambda_5 = -d_A - \gamma_A - \psi_A, \lambda_6 = -\mu - d_J - \gamma_J - \psi_J$$

The remaining eigenvalues are the solutions of

$$(-\lambda - \bar{d}_b - \bar{F}_b)(-\lambda - \bar{d}_b - \bar{\alpha}_b)(-\lambda - \bar{d}_b - \frac{\bar{B}\bar{\beta}_b}{\bar{d}_b}) - \bar{F}_b(-\bar{B}\bar{\beta}_b - \frac{\bar{B}\bar{\beta}_b \lambda}{\bar{d}_b}) = 0$$

or $$\lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3 = 0 \quad (3.2)$$

where

$$A_1 = 3\bar{d}_b + \bar{F}_b + \bar{\alpha}_b + \frac{\bar{B}\bar{\beta}_b}{\bar{d}_b} \quad (3.2a)$$

$$A_2 = 3\bar{d}_b^2 + 2\bar{d}_b \bar{F}_b + 2\bar{d}_b \bar{\alpha}_b + \bar{F}_b \bar{\alpha}_b + 2\bar{B}\bar{\beta}_b + \frac{\bar{B}\bar{\alpha}_b \bar{\beta}_b}{\bar{d}_b} \quad (3.2b)$$

$$A_3 = \frac{\bar{d}_b^2(\bar{d}_b + \bar{F}_b)(\bar{d}_b + \bar{\alpha}_b) + \bar{B}(\bar{d}_b^2 + (\bar{d}_b + \bar{F}_b)\bar{\alpha}_b)\bar{\beta}_b}{\bar{d}_b} \quad (3.3c)$$

We determine the conditions of $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and λ_6 to have negative real part by using Routh-Hurwitz criteria (Leah, E.K. ,1998).

$\det H_1 = A_1 > 0$ (3.3)

$\det H_2 = A_1 A_2 - A_3 > 0$ (3.4)

$\det H_3 = A_1 A_2 A_3 - A_3^2 > 0$ (3.5)

Condition (3.3) is always true because all terms in (3.2a) are positive.

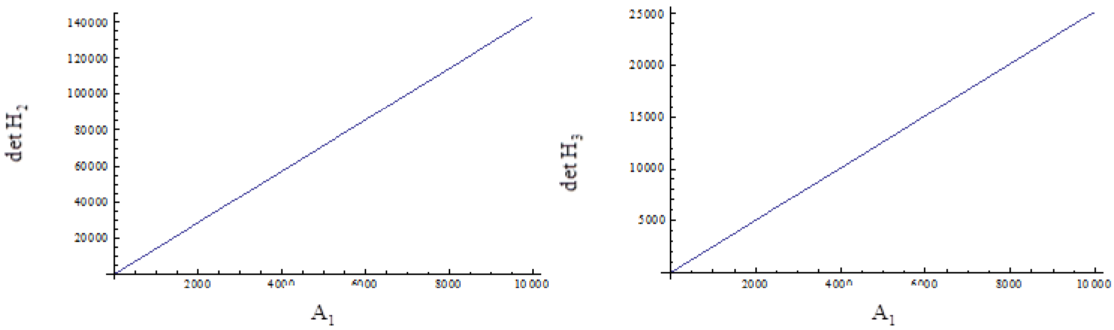


Fig.3 The parameter spaces for the disease-free equilibrium point which satisfy the Routh-Hurwitz criteria (3.4)-(3.5). The values of parameters are

$$\bar{B}=1500, \bar{d}_b=\frac{1}{100}, \bar{\alpha}_b=7, \bar{F}_b=\frac{1}{7}, \bar{\beta}_b=\frac{2.5}{200000}$$

ii) Endemic disease state $E_2=(S_b^*,E_b^*,I_b^*,S_J^*,E_J^*,I_J^*,R_J^*,S_A^*,E_A^*,I_A^*,R_A^*)$, the characteristic equation is

$$\begin{vmatrix}
-(\beta_{bJ}I_b+d_J+\mu)-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{bJ}S_J \\
\beta_{bJ}I_b & -(d_J+F_J+\mu)-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{bJ}S_J \\
0 & F_J & -(\gamma_J+\psi_J+d_J+\mu)-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma_J & -(d_J+\mu)-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu & 0 & 0 & 0 & -(\beta_{bA}I_b+d_A)-\lambda & 0 & 0 & 0 & 0 & 0 & -\beta_{bA}S_A \\
0 & \mu & 0 & 0 & \beta_{bA}I_b & -(d_A+F_A)-\lambda & 0 & 0 & 0 & 0 & \beta_{bA}S_A \\
0 & 0 & \mu & 0 & 0 & F_A & -(\gamma_A+d_A+\psi_J)-\lambda & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 & \gamma_A & -d_A-\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\beta}_b S_b-\bar{d}_b-\lambda & 0 & -\bar{\beta}S_b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{\beta}S_b & -\bar{F}_b-\bar{d}_b-\lambda & \bar{\beta}S_b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{F}_b & -\bar{\alpha}_b-\bar{d}_b-\lambda
\end{vmatrix}=0 \quad (3.6)$$

$$(-d_A-\lambda)(-d_A-F_A-\lambda)(-d_A-(\frac{\bar{B}\bar{F}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{d}_b}{\bar{\beta}_b})\beta_{bA}-\lambda)(\bar{d}_b^{\,3}+\bar{d}_b^{\,2}\bar{F}_b+\bar{d}_b^{\,2}\bar{\alpha}_b+\bar{d}_b\bar{F}_b\bar{\alpha}_b-\frac{\bar{B}\bar{d}_b^{\,2}\bar{F}_b\bar{\beta}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{B}\bar{d}_b\bar{F}_b^{\,2}\bar{\beta}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}$$
$$-\frac{\bar{B}\bar{d}_b\bar{F}_b\bar{\alpha}_b\bar{\beta}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{B}\bar{F}_b^{\,2}\bar{\alpha}_b\bar{\beta}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{2\bar{B}\bar{d}_b\bar{F}_b\bar{\beta}_b\lambda}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{B}\bar{F}_b^{\,2}\bar{\beta}_b\lambda}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{B}\bar{F}_b\bar{\alpha}_b\bar{\beta}_b\lambda}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-2\bar{d}_b\lambda^2-\bar{F}_b\lambda^2-\bar{\alpha}_b\lambda^2-$$
$$\frac{\bar{B}\bar{F}_b\bar{\beta}_b\lambda^2}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\lambda^3)(-d_J-\lambda-\mu)(-d_J-F_J-\lambda-\mu)(-d_J-(\frac{\bar{B}\bar{F}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{d}_b}{\bar{\beta}_b})\beta_{bJ}-\lambda-\mu)(-d_A-\gamma_A-\lambda-\psi_J)(-d_J-\gamma_J-\lambda-\mu-\psi_J)=0$$

$\lambda_1=-d_A$, $\lambda_2=-d_A-F_A$, $\lambda_3=-d_A-\gamma_A-\psi_A$, $\lambda_4=-d_J-\mu$, $\lambda_5=-d_J-F_J-\mu$, $\lambda_6=-d_J-\gamma_J-\mu-\psi_J$

$$\lambda_7=-d_J-\frac{\bar{B}\bar{F}_b\beta_{bJ}}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}+\frac{\bar{d}_b\beta_{bJ}}{\bar{\beta}_b}-\mu$$

λ_7 have negative real parts when

$$\frac{\bar{d}_b\beta_{bJ}}{\bar{\beta}_b}<d_J+\frac{\bar{B}\bar{F}_b\beta_{bJ}}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}+\mu$$

$$\lambda_8=-d_a-\frac{\bar{B}\bar{F}_b\beta_{bA}}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}+\frac{\bar{d}_b\beta_{bA}}{\bar{\beta}_b}$$

λ_8 have negative real parts when

$$\frac{\bar{d}_b\beta_{bA}}{\bar{\beta}_b}<d_a+\frac{\bar{B}\bar{F}_b\beta_{bA}}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}$$

The remaining eigenvalues are the solutions of

$$\bar{d}_b^{\,3}+\bar{d}_b^{\,2}\bar{F}_b+\bar{d}_b^{\,2}\bar{\alpha}_b+\bar{d}_b\bar{F}_b\bar{\alpha}_b-\frac{\bar{B}\bar{d}_b^{\,2}\bar{F}_b\bar{\beta}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{B}\bar{d}_b\bar{F}_b^{\,2}\bar{\beta}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{B}\bar{d}_b\bar{F}_b\bar{\alpha}_b\bar{\beta}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{B}\bar{F}_b^{\,2}\bar{\alpha}_b\bar{\beta}_b}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{2\bar{B}\bar{d}_b\bar{F}_b\bar{\beta}_b\lambda}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}$$
$$-\frac{\bar{B}\bar{F}_b^{\,2}\bar{\beta}_b\lambda}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\frac{\bar{B}\bar{F}_b\bar{\alpha}_b\bar{\beta}_b\lambda}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-2\bar{d}_b\lambda^2-\bar{F}_b\lambda^2-\bar{\alpha}_b\lambda^2-\frac{\bar{B}\bar{F}_b\bar{\beta}_b\lambda^2}{(\bar{d}_b+\bar{F}_b)(\bar{d}_b+\bar{\alpha}_b)}-\lambda^3=0$$

or $\lambda^3+A_1\lambda^2+A_2\lambda+A_3=0$

where

$$A_1 = 2\bar{d}_b + \bar{F}_b + \bar{\alpha}_b + \frac{\bar{B}\bar{F}_b\bar{\beta}_b}{(\bar{d}_b + \bar{F}_b)(\bar{d}_b + \bar{\alpha}_b)}$$

$$A_2 = \frac{2\bar{B}\bar{d}_b\bar{F}_b\bar{\beta}_b}{(\bar{d}_b + \bar{F}_b)(\bar{d}_b + \bar{\alpha}_b)} + \frac{\bar{B}\bar{F}_b^{\,2}\bar{\beta}_b}{(\bar{d}_b + \bar{F}_b)(\bar{d}_b + \bar{\alpha}_b)} + \frac{\bar{B}\bar{F}_b\bar{\beta}_b}{(\bar{d}_b + \bar{F}_b)(\bar{d}_b + \bar{\alpha}_b)}$$

$$A_3 = -\bar{d}_b(\bar{d}_b + \bar{F}_b)(\bar{d}_b + \bar{\alpha}_b) + \bar{B}\bar{F}_b\bar{\beta}_b$$

We can see that $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5$, λ_6,λ_7 and λ_8 have negative real parts. We use Routh-Hurwitz criteria

$$\det H_1 = A_1 > 0 \qquad (3.7)$$

$$\det H_2 = A_1A_2 - A_3 > 0 \qquad (3.8)$$

$$\det H_3 = A_1A_2A_3 - A_3^2 > 0 \qquad (3.9)$$

Condition (3.7) is always true because all terms are positive.

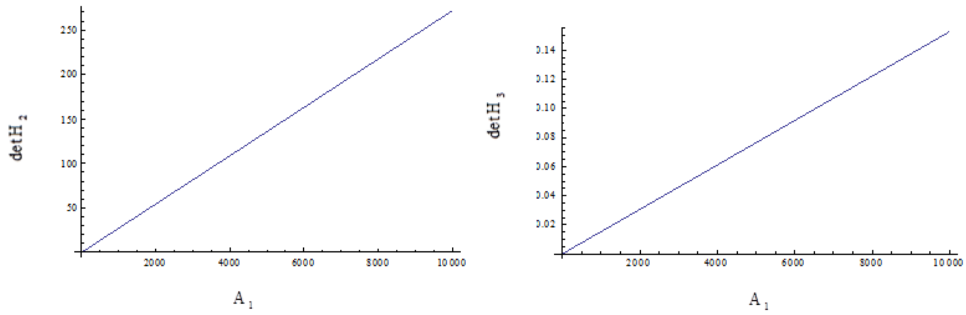


Fig4. The parameter spaces for the endemic equilibrium point which satisfy the Routh-Hurwitz criteria. The values of the parameter are $\bar{B} = 2{,}000$ **,** $\bar{d}_b = \frac{1}{20}$ **,** $\bar{F}_b = \frac{1}{7}$ **,** $\bar{\beta}_b = \frac{2.5}{200000}$ **,** $\bar{\alpha}_b = 4$ **.**

Numerical Results

The value of parameters in our model

Description	Parameter	Sample Values
Natural death rate of birds	$\bar{d}_b$	0.005 per day
death rate due to avian strain in birds	$\bar{\alpha}_b$	5 per day
rate at which susceptible bird	$\bar{\beta}_b$	0.000002 per day
incubation rate of avian influenza in birds	$\bar{F}_b$	0.142857 per day
Disease free		
bird inflow	$\bar{B}$	1,000 per day
Endemic		
bird inflow	$\bar{B}$	2,000 per day

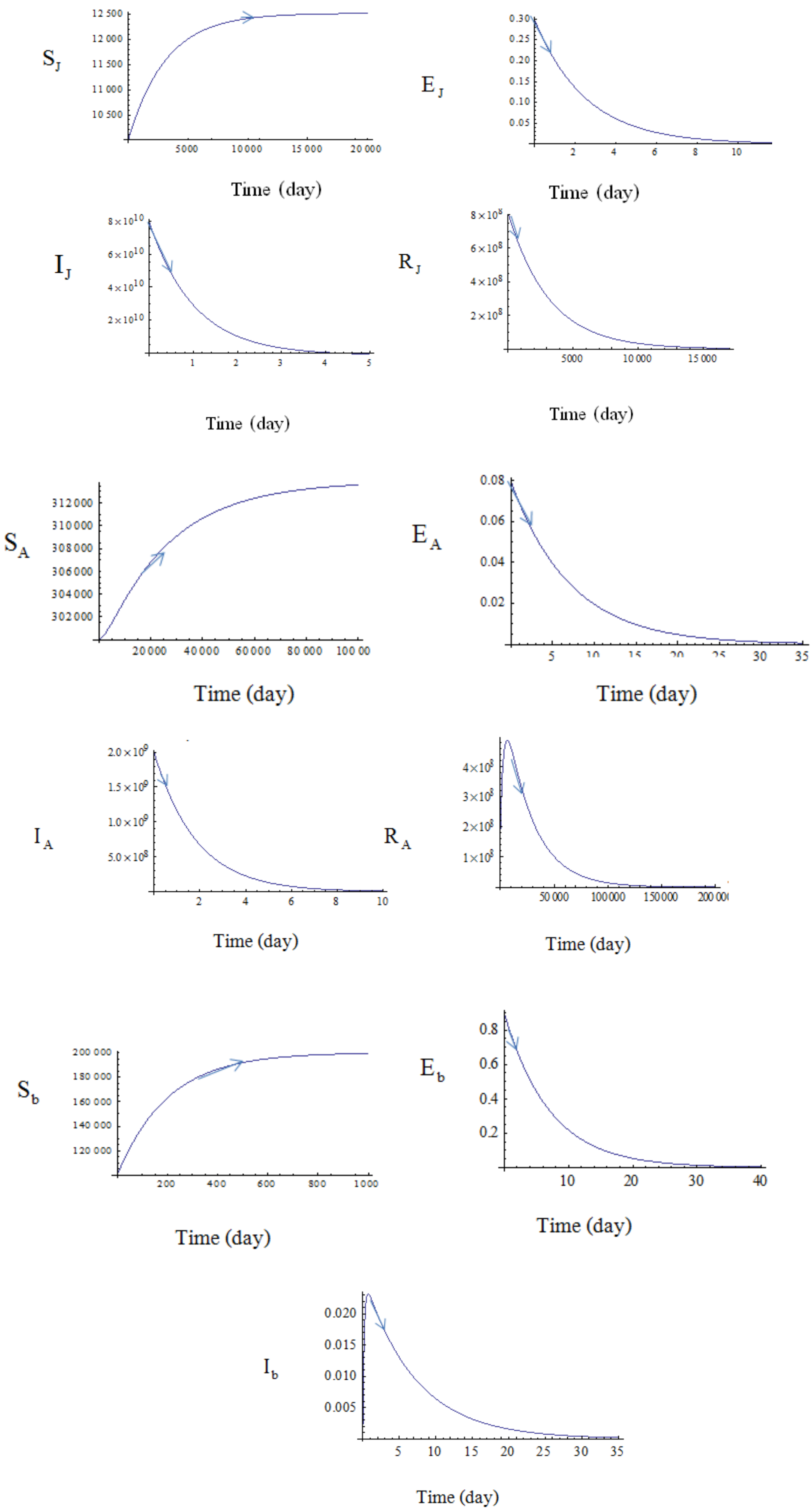

S J
Time (day)
E J
Time (day)
I J
Time (day)
R J
Time (day)
S A
Time (day)
E A
Time (day)
I A
Time (day)
R A
Time (day)
S b
Time (day)
E b
Time (day)
I b
Time (day)

Fig 5. Numerical solutions of our model for $R_0 < 1$ the parameters are

$\overline{B} = 1000, \overline{d}_b = 0.005, \overline{\alpha}_b = 5, \overline{\beta}_b = 0.000002, \overline{F}_b = 0.142857$ **and** $R_0 = 0.0422871$

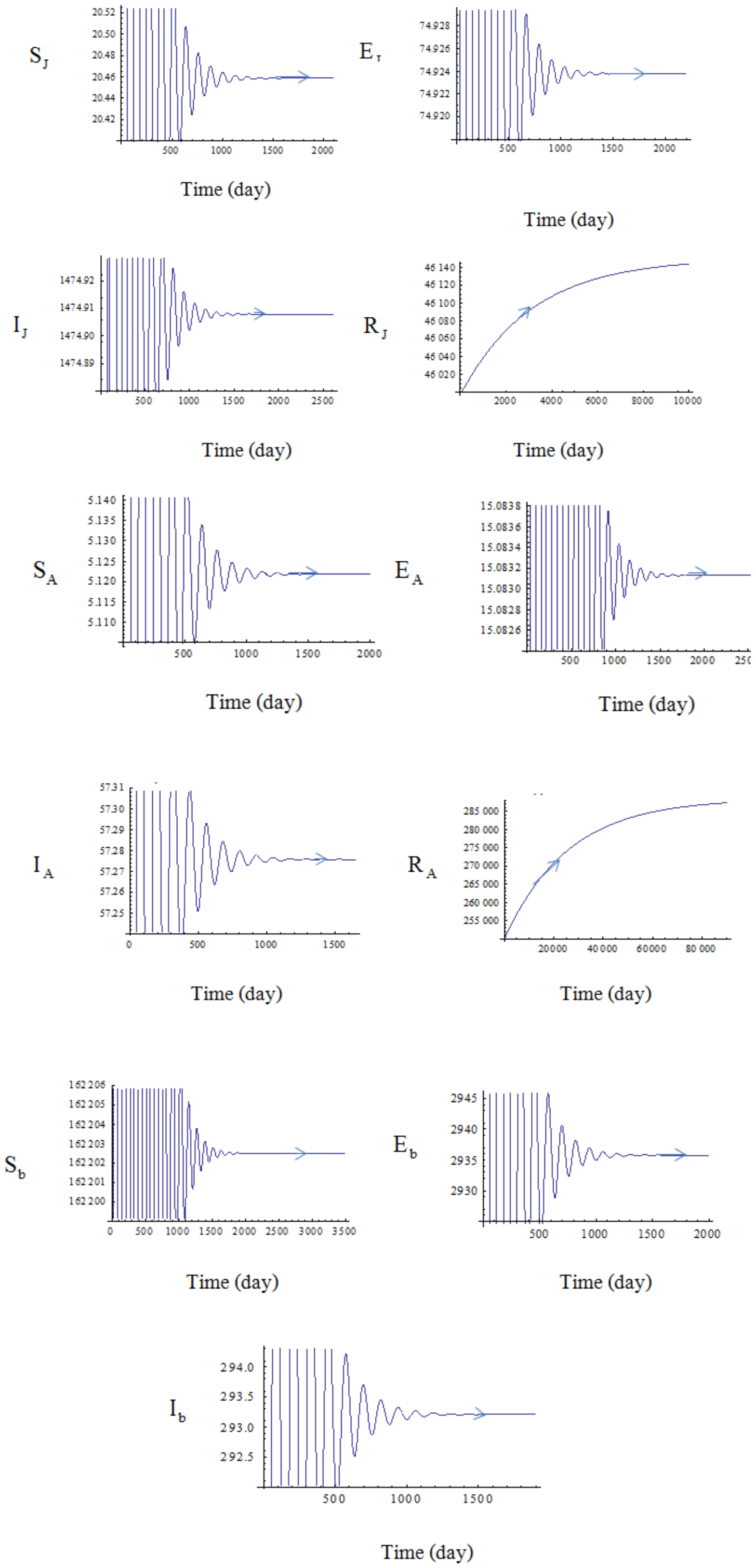


Fig.6 Numerical solutions of our model, for $R_0 > 1$ the parameters are

$\overline{B} = 2000, \overline{\beta}_b = 0.000025, \overline{d}_b = 0.005, \overline{\alpha}_b = 4, \overline{F}_b = 0.4$ **and** $R_0 = 6.4881$

DISCUSSION AND CONLUSION

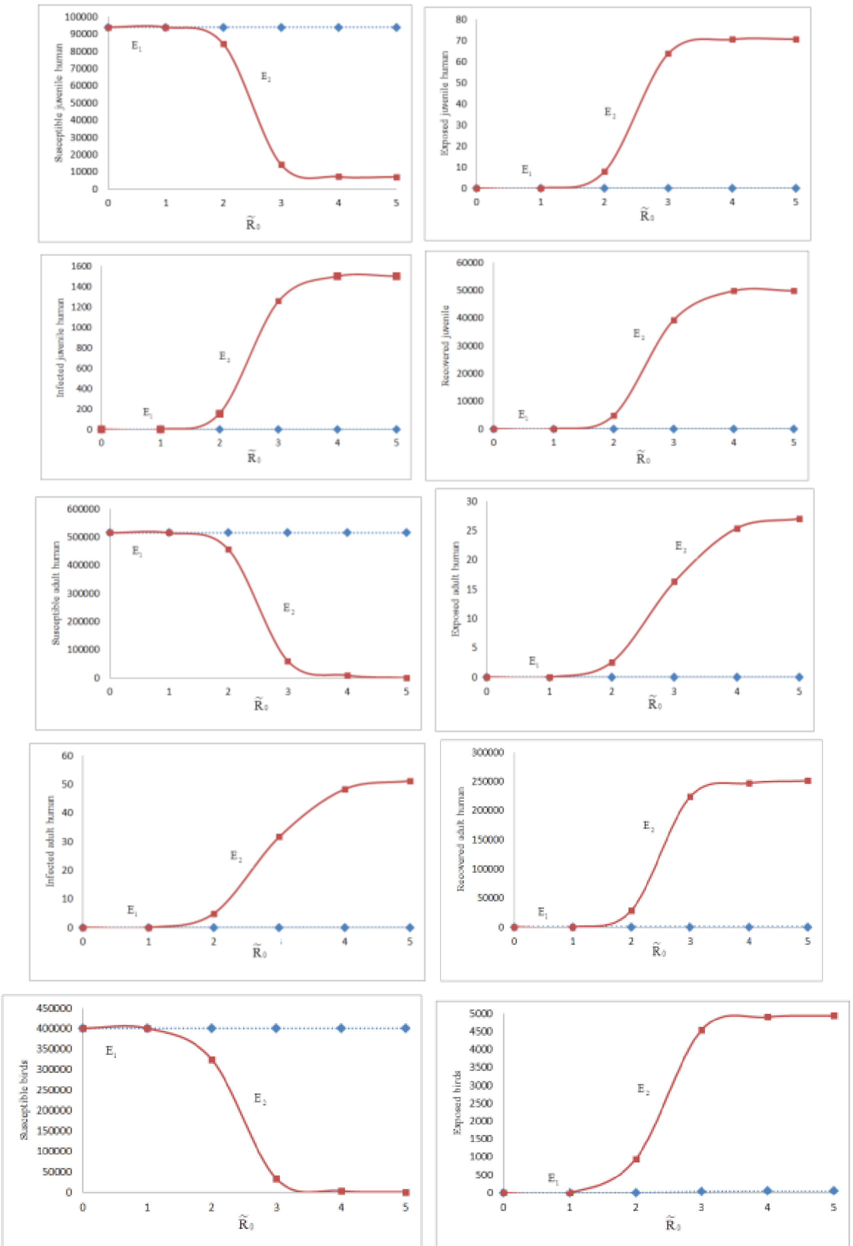
In this study, we constructed the mathematical model of avian influenza. The model equation is constructed for each human class. Standard dynamical modeling method is used for analyzing the behaviors of solution. The basic reproductive number is defined by R_0 where

$$R_0 = \frac{\bar{B}\bar{F}_b\bar{\beta}_b}{\bar{d}_b(\bar{d}_b + \bar{F}_b)(\bar{d}_b + \bar{\alpha}_b)}.$$

For $R_0 < 1$ the disease-free equilibrium state is stable and the endemic equilibrium is stable where $R_0 > 1$

From Fig.4, we can see that the solutions approach to the disease equilibrium state $E_1 = (12516, 0, 0, 0, 313893, 0, 0, 0, 200000, 0, 0)$ where $R_0 < 1$.

Fig.5 Time series solutions of susceptible, exposed, infected and recovered juvenile population and susceptible exposed infected and recovered adult population, susceptible, exposed and infected birds, where $R_0 < 1$



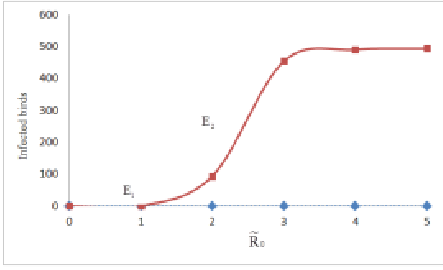


Fig.7 Bifurcation diagram of the solution of equation (1.1)-(1.11) the parameters are
$B = 2000, d_b = 0.005, F_b = 0.4, \alpha_b = 4$

The bifurcation diagrams of equation (1.1)-(1.11) are shown in fig.6. We can see that when $R_0 < 1$, E_1 will be stable and for $R_0 > 1$, E_2 will be stable. If the reproductive number is greater than one, the normalized susceptible exposed, infected, recovered populations, susceptible exposed infected, recovered adult humans. From the mathematical model of the avian influenza, controlling the epidemic model is effective and practical for the application of Mathematical to show numerical results of the mathematical model in accordance with the conditions of the outbreak and epidemic disease under the conditions without chronic conditions which could control the outbreak.

ACKNOWLEDGMENT

This work is supported by Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Thailand.

REFERENCES

Esteva, L., &Vargas, C. (1998). Analysis of a dengue disease Transmission model. Mathematical Biolosciences,150,131-151.

Government of Ontario (2006) Avian influenza: a guide to personal protective clothing and equipment for workers and employers working with or around poultry or wild birds. http://www.health.gov.on. ca/en/pro/programs/emb/avian/docs/avian_ppe_guide.pdf (Accessed 5 Feb 2013).

Leah, E.K. (1998). Mathematical Models in Biology. New York: Random House.

M. Gilbert, et al., Free-grazing Ducks and Highly Pathogenic Avian Influenza, Thailand. Emerg. Infect. Dis. 12 (2006), p. 227-234.

Mohamed, D., &Abdesslam, B. (2008).An Avian influenza mathematical model. Applied Mathematical Sciences 36,1749-1760.

N. Kung, et al., Risk for Infection with Highly Pathogenic Influenza A Virus (H5N1) in Chick-ens, Hong Kong, 2002. Emerg. Infect. Dis. 13 (2007), p. 412-418.

Nyuk Sian Chong , Jean Michel Tchuenche,&Robert J. Smith (2013). A mathematical model of avian influenza with half-saturated incidence.Theory Biosci. DOI 10.1007/s12064-013-0183-6.

World Health Organization (2012) H5N1 avian influenza: timeline ofmaj or events.http://www.who.int/influenza/human_animal_interface/avian_influenza/H5N1 avian_influenza_update.pdf (Accessed 5 Feb 2013).

World Health Organization (2007) Options for the use of humanH5N1 influenza vaccines and the WHO H5N1 vaccine stockpile. http://www. who.int/csr/ resources/publications/WHO_HSE_EPR_GIP_2008_1d.pdf (Accessed 5 Feb 2013).

World Health Organization (2011) New: WHO comment on the importance of global monitoring of variant influenza viruses. http://www.who.int/influenza/human_animal_interface/avian_influenza/h5n1-2011_12_19/en/index.html (Accessed 5 Feb 2013)

World Health Organization (2006) Avian influenza, including influenzaA (H5N1), in humans: WHO interim infection control guideline for health care facilities.https://www.premierinc.com/qualitysafety/toolsservices/safety/topics/influenza/downloads/07-whoai-inf-control-guide05-10-07.pdf (Accessed 5 Feb 2013).